CLOSED GEODESICS ON PRODUCTS OF MODULAR CURVES

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Abstract. Define $\widetilde{Y} \coloneqq \prod_{i=1}^{n} \Gamma_i\backslash\mathbf{PGL}_2(\mathbb{R})$ where Γ_i is a congruence lattice for all i, and denote by $Y \coloneqq \prod_i \Gamma_i\backslash\mathbf{PGL}_2(\mathbb{R})/\mathbf{PO}_2(\mathbb{R})$ the associated product of modular and Shimura curves. Let $A^{\Delta} < \prod_i \mathbf{PGL}_2(\mathbb{R})$ be the diagonal embedding of the group of diagonal matrices $A < \mathbf{PGL}_2(\mathbb{R})$. The periodic probability measures of A^{Δ} on $\widetilde{Y}$ are supported on lifts of closed geodesics from Y. Similar to the case of a single modular curve these periodic measures can be partitioned into finite arithmetic collections called packets – each packet is the projection of a single adèlic torus orbit. We determine the asymptotic distribution of a sequence of periodic measures on packets of closed geodesics on $\widetilde{Y}$, assuming the sequence satisfies a Linnik congruence condition at a single prime and the associated real quadratic fields have no exceptional Landau-Siegel zero. This problem is the archimedean analogue of the Michel-Venkatesh mixing conjecture.

The proof builds upon the author's recent work on joint equidistribution of CM points. In comparison to the CM case the non-compactness of the archimedean torus is a source of several obstacles. The foremost is that the shifted convolution sums that arose in the treatment of CM points are replaced by more general sums that do not have the shifted convolution structure. We also remove en route the restriction on the conductors in the CM case.